## Exercises for Chapter 4

4.1 Doty, Gillian, LeBaron, and Sawaya studied the effects of the number of Automatic Guided Vehicles, AGV's(1-3), Speed of AGV's (120/100-180/150 ft./min.) and the pick up and drop off times of AGV's upon the total throughput time in a castings factory. A $2^{3}$ design with replicated center points was used, and the data was generated by the ProModel Simulation program. The data is listed below:

| $\begin{gathered} \mathrm{X}_{1}= \\ \text { Number of } \\ \text { AGV's } \end{gathered}$ | $\mathrm{X}_{2}=$ <br> Speed of <br> AGV's | $\begin{gathered} \mathrm{X}_{3}= \\ \text { Pick/Drop } \\ \text { time } \\ \hline \end{gathered}$ | Throughput |
| :---: | :---: | :---: | :---: |
| - | - | - | 180184181 |
| + | - | - | 202204207 |
| - | + | - | 199194195 |
| + | + | - | 206213210 |
| - | - | + | 190186184 |
| + | - | + | 208208209 |
| - | + | + | 196203204 |
| + | + | + | 215210195 |
| 0 | 0 | 0 | $\begin{array}{lll} 194 & 201 & 195 \\ 204 & 197 & 203 \end{array}$ |

a) Calculate the mean and variance, $\mathrm{s}^{2}$, of the replicate responses for each run, and calculate the pooled standard deviation $\mathrm{s}_{\mathrm{p}}$.
b) Do a graphical analysis by plotting the mean responses at each run on a cube.
c) Calculate the main effects, two-way interactions, the three-way interaction, and the curvature effect.
d) Calculate the standard error of an effect, $\mathrm{s}_{\mathrm{E}}$, the standard error of the curvature effect, $\mathrm{s}_{\mathrm{C}}$,
e) Determine which effects and interactions are statistically significant. Is curvature significant?
4.2 A study was performed to investigate the effects of iron deficiency in diets of rats on their activity. The measure of activity, Y, was the time spent in vertical movement (seconds per 15 -minute interval). Theresponsewas measuredautomatically and continuously using a rat cage with infrared beams connected to a computer monitoring system. The response recordedwasthe average of 4015 -minute readingstakenduring 10 hours of lightor 10 hours of darkness (with 2 hours in between). The factors studiedwere:

| $\mathrm{X}_{1}:$ | Iron in diet | $-=$ Deficient |
| :--- | :--- | :--- |
| $\mathrm{X}_{2}:$ | Age of rat | $-=3$ weeks old |
| $\mathrm{X}_{3}:$ | Time of day | $-=$ day (light) |
|  | $+=6$ weeks old |  |
|  |  | $+=$ night $($ dark $)$ |


| Iron in Diet | Age, weeks | Day or <br> Night |  | Response, YLitter of Rats |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{X}_{1}$ | $\mathrm{X}_{2}$ | $\mathrm{X}_{3}$ | 1 | 2 | 3 | 4 |
| -1 | -1 | -1 | 4.1 | 9.0 | 3.7 | 5.2 |
| 1 | -1 | -1 | 2.6 | 8.4 | 4.3 | 7.5 |
| -1 | 1 | -1 | 4.9 | 7.2 | 3.8 | 3.3 |
| 1 | 1 | -1 | 3.1 | 8.8 | 4.8 | 4.9 |
| -1 | -1 | 1 | 10.3 | 14.2 | 12.7 | 12.8 |
| 1 | -1 | 1 | 10.9 | 18.5 | 12.8 | 15.0 |
| -1 | 1 | 1 | 5.6 | 12.1 | 4.8 | 6.3 |
| 1 | 1 | 1 | 10.2 | 15.9 | 12.8 | 10.3 |

Four rats were used for each set of conditions, each from a different litter of rats. The data collected are given above.
a) Treating the rats as pure replicates, analyze the data to determine which factors and interactions are statistically significant. Summarize your results in equation form.
b) Treating the rat litters as blocks, reanalyze the datato determinewhich factors and interactions are statisticallysignificant. Note:Use the interactions of blocking variables with thefactors studied to estimate $\mathrm{S}_{\mathrm{E}}$. Summarize your results in equation form.
c) What was the impact of treating the litters as blocks on your analysis?
4.3 Suppose that in the experiment described in Exercise 3.2, it was desired to detect any main effect or interaction that would effect the percent shrinkage of injection molded parts by $10 \%$ or more. Factors with effects this large could then be controlled to reduce shrinkage and save considerable expenses in assembly. If the historical standard deviation in percent shrinkage from replicate parts molded under the same conditionswas $\sigma=4 \%$, how many replicates of the $2^{3}$ design you created for Exercise 3.2 are required?
4.4 The following exercises are based on the experiment on milling aluminum in Exercise 3.3.
a) Assuming the historical standard deviation of the surface finish of milled aluminum was $\sigma=12.5$, what is the smallest effect that you coulddetect with $95 \%$ confidence, using three replicates of the full design shown in Exercise 3.3.
b) Using the data from Exercise 3.3, developa predictionequation using the significant effects and or interactions.
c) Using the predictionequation youdeveloped in Part(b), predict the value of surface finish with Spindle Speed $=1200$ RPM, Feed Rate $=10$ IPM, and Depth of Cut $=0.020$.
4.5 List the experiments in randomized order for
a) A $2^{3}$ design
b) A $2^{3}$ design run in two blocks
c) $\mathrm{A} 2^{3}$ design run in four blocks
4.6 Router bits are used to smooth the edges of printed circuit boards. When a bit wears out, dust is created that clings to the edge surface of the circuit boards. This requires an extra cleaning step, or creates difficulty while sliding the boards into slots during assembly. An experiment is to be designed to test the effect of two factors attwo levels upon the useable life of routerbits. The factors are $\mathrm{X}_{1}=$ Router RPM, and $\mathrm{X}_{2}=$ feed rate. An experiment will consist of setting the RPM and feed rate, then routing with one bit until dust begins to form indicating the bit is worn. Router bits have four positions, and thus four experiments can be made with each bit. It is known that useable life differs between bits (i.e., a known lurking variable).
a) Set up a 16 runblocked design to test the effects and interactions of the two factors. Show the run order of the experiments, and explain how you would calculate the error of an effect.
b) Set up a 32 runblocked design to test the effects and interactions of the two factors. Show the run order of the experiments, and explain how you would calculate the error of an effect.
4.7 In the study dealing with a solar water heating system that was given in Exercise 3.4, one additional run of the computer model was made at the center point, where the coded factors levels were $\mathrm{X}_{1}=0$ (the total daily insolation), $\mathrm{X}_{2}=0$ (the tank storage capacity), $\mathrm{X}_{3}=0$ (the water mass flowratethroughthe absorber), and $X_{4}=0$ (the intermittency of the inputfrom solarradiation). The responses at this condition were $\mathrm{Y}_{1}=41.6$ (the collection efficiency) and $\mathrm{Y}_{2}=100.0$ (the energy deliveryefficiency). This additional datawould allowfor checking the adequacy of a simpler linear approximation model to be determined.
a) Write a simple linear prediction model for each response using the effects that appear important on the plots you constructed in Exercise 3.4.
b) Determine an estimate of $\mathrm{s}_{\mathrm{E}}$ from the Normal plot of effects you made in Exercise 3.4.
c) Calculate the constant that the ratio $\mathrm{s} \delta \mathrm{s}_{\mathrm{E}}$ would equal for this experiment.
d) With reference to the constant calculated in Part (c), does curvature appear to play an important role?
4.8 Four different fabrics defined by the \% rayonandweavestyle were compared on a Martindale wear tester for suitability for automobile seats. The wear tester can compare four samples of cloth in a single run (block). The weight loss in milligrams from four runs were measured and the following results recorded:

| Run Order or |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| TesterPos. | Block | \%Rayon | Weave Style | Weight loss |
| 1 | 1 | - | - | 36 |
| 4 | 1 | + | - | 36 |
| 3 | 1 | - | + | 38 |
| 2 | 1 | + | + | 30 |
| 3 | 2 | - | - | 17 |
| 2 | 2 | + | - | 26 |
| 4 | 2 | - | + | 18 |
| 1 | 2 | + | + | 17 |
| 3 | 3 | - | - | 30 |
| 1 | 3 | + | + | 41 |
| 4 | 3 | - | + | 39 |
| 2 | 3 | - | - | 34 |
| 2 | 4 | - | + | 30 |
| 3 | 4 | - | + | 38 |
| 1 | 4 | + |  | 40 |
| 4 | 4 |  |  | + |

a) Determine if there are any significant effects of \% rayon, weave style, or their interaction.
b) Was the blocking of experiments useful?
4.9 A design engineer wished to determine the relationship between the type of fastener, thread type, hole size, size of the hole relative to its tolerance, and the clamp load of a fastener constant torque(response).Each experimentconsistedofinstalling a fastenerand tighteningit to the specified torque then measuring the clamp load between the two sheets being fastened. The factors and levels are listed below:

| Factor | - | + |
| :---: | :---: | :---: |
| 1. Type Fastener | Standard | Self Threading |
| 2. Thread | Rolled | Tapped |
| 3. Hole Size | 1/4 | 3/8 |
| 4. Hole Relative | Low end of | High end of |
| to Tolerance | Tolerance | Tolerance |

a) List the experiments in a worksheet form and indicate the order that they should be performed, in order to examine the effects.
b) Differences in clampload of 10 psi are important (becausereduction of 10 psi may cause failure of the final product). From previous control charts produced using one type of fastener, thread type, and holesize the standard deviation of clampload was estimated to be $\sigma=6.07$. How many replicates do you recommend for this experiment?
c) If hole relative to tolerance was hard to vary from experiment to experiment, and it would be much more convenient to run the experiments in groups with constant hole relative to tolerance, how would you change the experimental design? List a new worksheet and show the order in which experiments are conducted.
4.10 For the scenario given in Exercise $3.3\left(\mathrm{n}_{\mathrm{F}}=3 \times 2^{3}=24, \mathrm{~s}=12.7\right)$ :
a) Were an appropriate number of experiments performed if $\delta=15$ ? If not, what would be an appropriate number of experiments?
b) For the design that was used $\left(\mathrm{n}_{\mathrm{F}}=24\right)$ what $\delta$ were you fairly sure ( $95 \%$ ) of detecting? (Note: For simplicity, just use Equation 4.2.)
c) Summarize the result of your analysis of Exercise 3.4 (conducted for Chapter 3
homework) in equation form. Reminder: include only significant effects and interactions.
4.11 Sausage casings are extruded from a gel batch made from collagen (the underside of cow hides). The gel batch was made by soaking the collagen in extremely hot water for a prolonged period. Next, the gel is extruded into a tube and fixed by inflating it with ammonia. It was desirable to make the diameter of the extruded tube as consistent as possible. Therefore an experiment was set up to test factors in both the gel-making stage as well as the extrusion stage to see their effect on diameter consistency. Consistency was measured as the log of the variance of several diameter measurements taken along the length of the tube. Belowis a list of the factors and levels:

| $\quad$ Factor | - | + |
| :--- | :--- | :--- |
| 1. Gel batch soaking time, $\mathrm{X}_{1}$ | 48 hrs | 72 hrs |
| 2. Extrusion pump pressure, $\mathrm{X}_{2}$ | 30 psi | 45 psi |
| 3. Ammonia concentration, $\mathrm{X}_{3}$ | low | high |

Since Factors 2 and 3 could be varied within a gelbatch, but Factor 1 could only be varied between gel batches, four different gel batches were made in order to replicate Factor 1. The first two gel batches were labeled Replicate 1 and the second two as Replicate 2. The data from the experiment are shown below:

| Run | Rep | Gel | $\mathrm{X}_{1}$ | $\mathrm{X}_{2}$ | $\mathrm{X}_{3}$ | $\log \left(\mathrm{~s}^{2}\right)$ |
| :---: | :--- | :--- | :--- | :--- | :--- | :--- |
| 1 | 1 | 1 | - | - | - | 28.2359 |
| 2 | 1 | 1 | - | + | - | 26.5620 |
| 3 | 1 | 1 | - | - | + | 15.5834 |
| 4 | 1 | 1 | - | + | + | 26.4054 |
| 5 | 1 | 2 | + | - | - | 37.0237 |
| 6 | 1 | 2 | + | + | - | 58.9393 |
| 7 | 1 | 2 | + | - | + | 30.8361 |
| 8 | 1 | 2 | + | + | + | 51.4071 |
| 9 | 2 | 3 | - | - | - | 32.5547 |
| 10 | 2 | 3 | - | + | - | 42.3253 |
| 11 | 2 | 3 | - | - | + | 20.4696 |
| 12 | 2 | 3 | - | + | + | 28.4434 |
| 13 | 2 | 4 | + | - | - | 58.6681 |
| 14 | 2 | 4 | + | + | - | 73.3822 |
| 15 | 2 | 4 | + | - | + | 57.0995 |
| 16 | 2 | 4 | + | + | + | 71.9670 |

a)Perform analysis of this datato determine if there are significant effects of factors $X_{1}-X_{3}$ or any of their two-way or three-way interactions.
b) Write a sentence or two interpreting each significant effect or interaction you find.
c) Make a recommendation of the conditions that will result in the most consistent casing.
4.12 For the scenario given in Exercise 3.4 ( $\mathrm{n}_{\mathrm{F}}=3 \times 2^{3}=24, \mathrm{~s}=4.5$ ):
a) Were an appropriate number of experiments performed if $\delta=10$ ? If not, what would be an appropriate number of experiments?
b) For the design that was used $\left(\mathrm{n}_{\mathrm{F}}=24\right)$ what $\delta$ were you fairly sure $(95 \%)$ of detecting?
(Note: For simplicity, just use Equation 4.2.)
c) Summarize the result of your analysis of Exercise 3.4 (conducted for Chapter 3 homework) in equation form. Reminder: include only significant effects and interactions.
4.13 Let us pretend theatwhile Phil Jacksonwas coaching Michael Jordon for the Chicago Bulls (of the National Basketball Association), he was interested in figuring out what factors influenced Michael's performance during a basketball game. He(Phil Jackson) suspected that the number of McJordonburgers Michael consumed the aftemoonbefore the game, the hours of sleepMichael got the night before the game, and the number of glasses of JordonadeMichaeldrank the evening of the game, all influenced how many points Michael scored during the game. Phil heard, via the grapevine, that you were a statistical whiz, and he called you to design an experiment for the study. You recommended a tried and true factorial design plus center points, and Phil managed to collect the following data:

| $\begin{gathered} \mathrm{X}_{1}=\text { Burgers } \\ -=1 \\ 0 \end{gathered}$ | $\begin{aligned} & \mathrm{X}_{2}=\text { Sleep } \\ & -=6 \text { hours } \\ & 0=7 \text { hours } \\ & +=8 \text { hours } \end{aligned}$ | $\begin{aligned} \mathrm{X}_{3} & =\text { Jordonade } \\ - & =2 \text { glasses } \\ 0 & =3 \text { glasses } \\ + & =4 \text { glasses } \end{aligned}$ | $\mathrm{Y}=$ Performance (Points scored in the basketball game) |
| :---: | :---: | :---: | :---: |
| - | - | - | 25 |
| + | - | - | 31 |
| - | + | - | 49 |
| + | + | - | 53 |
| - | - | + | 37 |
| + | - | + | 14 |
| - | + | + | 29 |
| + | + | + | 38 |
| 0 | 0 | 0 | 30,27,31 |

Do any of the factors have an impact on Michael's performance? Back up your conclusions with the appropriate calculations.
4.14 The following is a continuation of Exercise 3.6, which is based on the performance of a gas chromatograph.
(a) Develop a prediction equation based on the significant effects.
(b) Using the prediction equation from Part (a), determine the conditions (within the range of the experimental data) whichmaximize the peak arearesponse. (Hint: Use a spreadsheet solver function.)
4.15 The following is a continuation of Exercise 3.8, which studied prototype replacement hip joints.
(a) Develop a prediction equation based on the significant effects for both force to dissociate, and range of motion.
(b) Using your two prediction equations from Part (a), try to locate conditions (within the range of the experimental data) which will maximize force to dissociate, while maintaining a range of motion that is at least 49 degrees.

